Occasional Delayed Feedback Control for Piecewise-Smooth Systems

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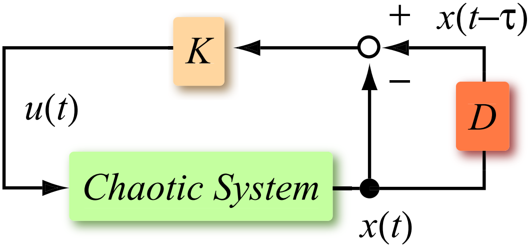
Background

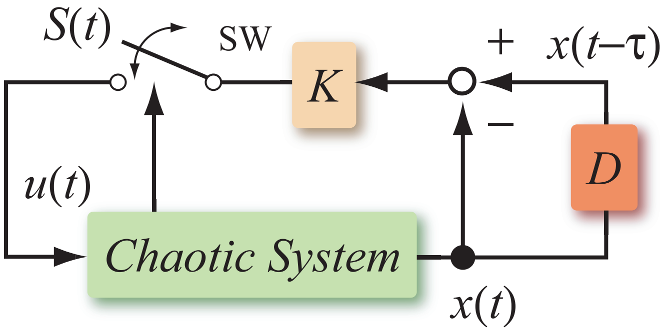
Controlling chaos — To stabilize an unstable periodic orbit (UPO) in given chaotic attractor; two major methods

1. **OGY method — based on linear control theory**
2. **delayed feedback control(DFC)**



ODFC 1/2

Occasional delayed feedback control(ODFC)— our original method



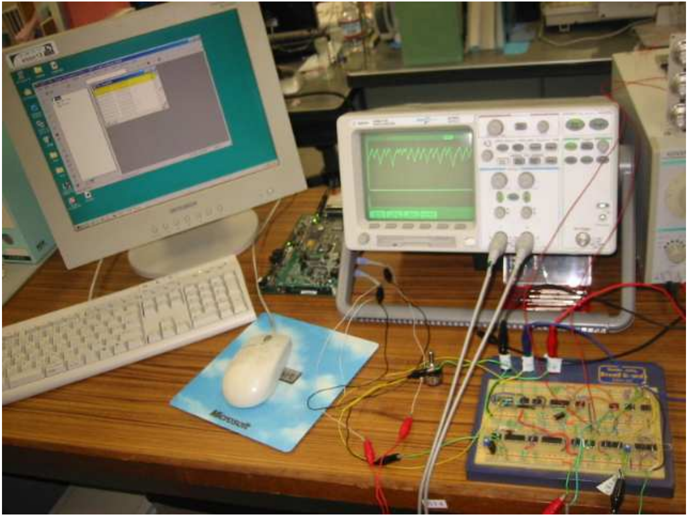
The original trial of controlling chaos for switched dynamical system

ODFC 2/2

$$\frac{d\mathbf{x}}{dt} = \begin{cases} \mathbf{f}(\mathbf{x}) + \mathbf{K}(\hat{\mathbf{x}} - \mathbf{x}) & \textbf{if} \quad \mathbf{x} \in M \\ \mathbf{f}(\mathbf{x}) & \textbf{if} \quad \mathbf{x} \notin M \end{cases} \tag{1}$$

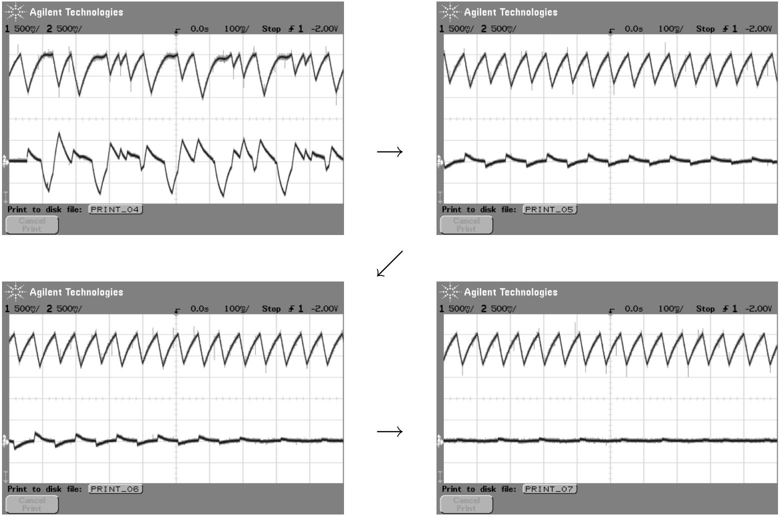
- **Very simple**
- **Not always use delayed control input.**
- **Sometime the total control energy is less than normal DFC.**

ODFC; physical implementation 1/2



1D chaos circuit + DSP controller

ODFC; physical implementation 2/2



Problems

Effectivity of ODFC has been only confirmed in a 1D chaotic system.

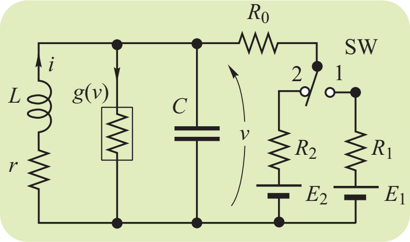
- **wasn't it by chance ?**
- **wasn't it applicable to other systems ?**

$$\Downarrow$$

How are cases of higher-dimensional switched dynamical systems ?

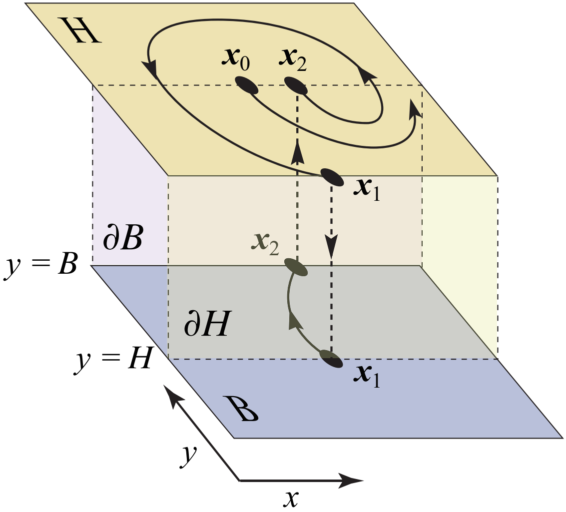
Alpazur Oscillator

2 dimensional autonomous system

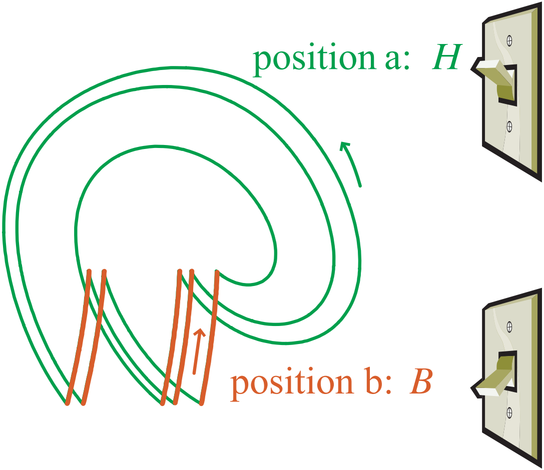


$$\begin{aligned} \frac{dx}{dt} &= -kx - y \\ \frac{dy}{dt} &= x + y - \frac{y^3}{3} - \begin{cases} g_1 y - B_1 & \textbf{if } \mathbf{x} \in H \\ g_2 y - B_2 & \textbf{if } \mathbf{x} \in B \end{cases} \end{aligned} \tag{2}$$

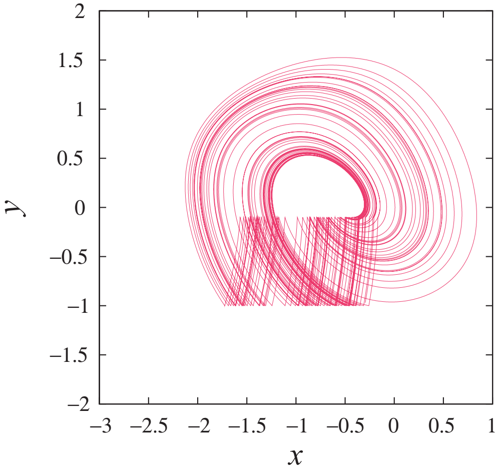
Solution flow



Switching action

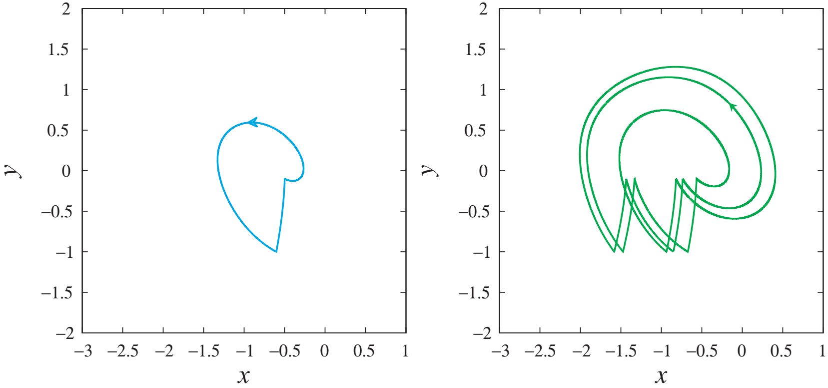


Chaos



$k = 0.1,\ g_1 = 0.2,\ g_2 = 2.0,\ H = -1.0,\ B = -0.1,\ B_1 = 5,$
$B_2 = 0.5$

Embedded UPOs and switching patterns



ODFC settings

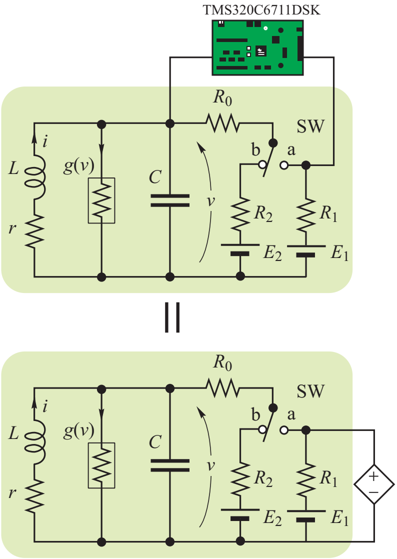
Control input: $\mathbf{u}(t) = \mathbf{K}(\mathbf{x}(t) - \mathbf{x}(t-\tau))$.
Controlled system:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) + \mathbf{u}(t)$$

Two patterns of ODFC implementation:

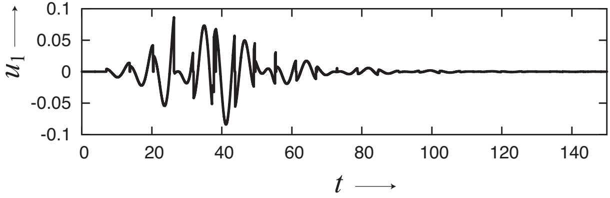
1. **add $\mathbf{u}(t)$ if the switch is turned into 'a' position. Pattern 1, otherwise no control.**
2. **add $\mathbf{u}(t)$ if the switch is turned into 'b' position. Pattern 0, otherwise no control.**

Circuit diagram

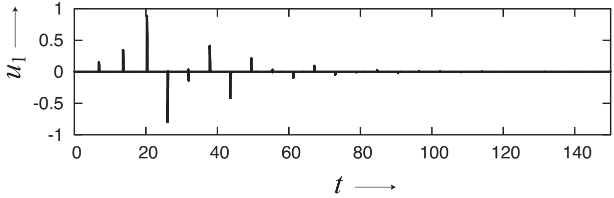


Time responses of $u(t)$.

Figures in the proceeding:



(a): pattern 1: $K = (0.15, 0.15)$, $\mathbf{x}(0) = (-0.5, -1)$.



(b): pattern 0: $K = (5, 5)$, $\mathbf{x}(0) = (-0.5, -1)$.

Multi-input multi-output controller.

Single-input single-output

To realize 'a voltage controlled voltage source.'

- **We choose a gain matrix**

$$\mathbf{K} = \begin{pmatrix} k_{11} & 0 \\ 0 & k_{22} \end{pmatrix}$$

as single voltage sensing: $\Rightarrow$ **put** $k_{22} = 0$

- **Choose pattern 0: few chances can control a UPO !**

RESULT: Only 2.8 % chance can can control UPO and its saves about 82 % energy compared with pattern 1.

Conclusions

ODFC for 2-dimensional piecewise-smooth system; Alpazur oscillator:

- **ODFC can stabilize one of UPOs.**
- **In the steady state, 97.2 % is uncontrolled. $\Rightarrow$ It can save some kinds of energy.**

Future problems:

- **How is other UPOs ?**
- **Stability analysis.**