Bifurcation and Chaos in the Extended BVP Oscillator

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Coupled oscillators

- practical industrial applications models
- biological activities
Coupled oscillators

- practical industrial applications models
- biological activities

Decomposite a complex nonlinear dynamics into unit oscillators and their connections
Coupled oscillators

- practical industrial applications models
- biological activities

Decompose a complex nonlinear dynamics into unit oscillators and their connections

\[ \Downarrow \]

a reduced dynamical system with symmetry

- synchronization
- global/local bifurcations
Previous studies

Resistively coupled BVP oscillators

- Symmetrical connected oscillators have much variety of synchronization modes of limit cycles

- Asymmetrical coupling induces Chaos!
Previous studies

Resistively coupled BVP oscillators

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- but **do not have chaos** within reasonable parameter range.
Previous studies

Resistively coupled BVP oscillators

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- but do not have chaos within reasonable parameter range.
- since symmetrical properties rather build “mild” dynamics.
Previous studies

Resistively coupled BVP oscillators

- Symmetrical connected oscillators have much variety of synchronization modes of limit cycles
- but **do not have chaos** within reasonable parameter range.
- since symmetrical properties rather build “mild” dynamics.
- asymmetrical coupling induces **Chaos**!
Single BVP oscillator

Circuit equation:

\[ \frac{C}{v} \frac{dv}{dt} = g(v) \]

\[ \frac{L}{i} \frac{di}{dt} = v + E \]
Single BVP oscillator

Circuit equation:

\[ C \frac{dv}{dt} = -i - g(v) \]
\[ L \frac{di}{dt} = v - ri + E \]
Bifurcations in single BVP

\[ k \]

\[ \gamma \]

(a)

(b)

(c)

(d)

(e)

G

h_1

h_2

Oscillatory

Bifurcation and Chaos in the Extended BVP Oscillator
Resistively coupled BVP oscillators

\[ \begin{align*}
L & \quad C \\
r_1 & \quad v_1 \\
R & \quad g(v_1) \\
L & \quad C \\
r_2 & \quad v_2 \\
R & \quad g(v_2)
\end{align*} \]
Resistively coupled BVP oscillators

\[ g(v_1)Lv_1Cg(v_2)Lv_2Cg(v_1)Lv_1Cg(v_2)Lv_2C \]
Resistively coupled BVP oscillators
Resistively coupled BVP oscillators
$\nu$-$\nu$ coupled BVP oscillators

In this talk... 

Reducing the $v$-$i$ coupled circuit

$$g(v_1) \quad C \quad L \quad R \quad g(v_2)$$

Removing $g(v_2)$, letting $r_2 = 1$, $R = 0$.
In this talk . . .

Reducing the \( \nu-\dot{i} \) coupled circuit

removing \( g(\nu_2) \), letting \( r_2 = \infty \), \( R = 0 \),
In this talk . . .

Reducing the $v$-$i$ coupled circuit

removing $g(v_2)$, letting $r_2 = \infty$, $R = 0$, 
Extended BVP oscillator

This circuit have already shown as one of all combinations of $L, C, r, g(v)$. [Chua et al, 1992]
Extended BVP oscillator

This circuit have already shown as one of all combinations of $L$, $C$, $r$, $g(v)$. [Chua et al, 1992] But not mentioned these dynamics.
Extended BVP oscillator

This circuit have already shown as one of all combinations of $L$, $C$, $r$, $g(v)$. [Chua et al, 1992]
☞ But not mentioned these dynamics.
☞ Naturally induced from coupled system!
Circuit equation

\[
\begin{align*}
C \frac{dv_1}{dt} &= -i - g(v_1) \\
C \frac{dv_2}{dt} &= i - \frac{v_2}{r} \\
L \frac{di}{dt} &= v_1 - v_2
\end{align*}
\]
Circuit equation

\[
\begin{align*}
C \frac{dv_1}{dt} &= -i - g(v_1) \\
C \frac{dv_2}{dt} &= i - \frac{v_2}{r} \\
L \frac{di}{dt} &= v_1 - v_2
\end{align*}
\]

\[g(v) = -a \tanh bv, \quad \tau = t/\sqrt{LC},\]

\[
\begin{align*}
\gamma &= ab \sqrt{\frac{L}{C}}, \quad \delta = \frac{1}{r} \sqrt{\frac{L}{C}} \\
x &= \frac{v_1}{a} \sqrt{\frac{C}{L}}, \quad y = \frac{v_2}{a} \sqrt{\frac{C}{L}}, \quad z = \frac{i}{a}.
\end{align*}
\]
Normalized equation

\[ \dot{x} = -z + \tanh \gamma x \]
\[ \dot{y} = z - \delta y \]
\[ \dot{z} = x - y \]

Equivalently,

\[ \ddot{x} + \alpha(x) \dot{x} + \beta(x, \dot{x}) \dot{x} + \delta x - \tanh \gamma x = 0 \]

where,

\[ \alpha(x) = \delta - \gamma \text{sech}^2 \gamma x \]
\[ \beta(x, \dot{x}) = 2 + 2\gamma^2 \text{sech}^2 \gamma x \tanh \gamma x \dot{x} - \delta \gamma \text{sech}^2 \gamma x \]
\[ g(v) = -a \tanh bv \]

\(\gamma\)-sensitivity:
The output of an inverter driving this circuit can realize $-a \tanh bv$. 
Experimental measurement

\[ i \text{ [mA]} \]

\[ v \text{ [V]} \]

\( \gamma \) can be controlled by \( R_c \).

\( R_c = 1000 \)
\( R_c = 500 \)
\( R_c = 200 \)
\( R_c = 100 \)
\( R_c = 0 \)
Bifurcation diagram

(A) Oscillatory
(B) Non-oscillatory
(C) Oscillatory
(D) Non-oscillatory
(E) Oscillatory

\( h_1 \)
\( d + G_1 \)
\( I \)
\( h_2 \)
\( G_2 \)
\( G_3 \)
\( Pf \)

Chaos

\( d + h_0 \)

\( \delta \)

\( \gamma \)
Bifurcation and Chaos in the Extended BVP Oscillator
Double scroll

\[ x-y \]  \[ y-z \]  \[ x-z \]
Perspectives
Perspectives
Laboratory experiments

By changing an serial resistance in $g(v)$, $r = 467[\Omega]$. 
Jack-in-the-box phenomenon
Jack-in-the-box phenomenon (2)
Long transient of jack-in-the-box

\[(x_0, y_0, z_0) = (3, 5, 8)\]
Long transient of jack-in-the-box

\[(x_0, y_0, z_0) = (3, 5, 8)\]
Long transient of jack-in-the-box

\( (x_0, y_0, z_0) = (3.00001, 5, 8) \)
Long transient of jack-in-the-box

\[ (x_0, y_0, z_0) = (3.00001, 5, 8) \]
Generation of jack-in-the-box

Wreck of a limit cycle

shrink
Basin boundary in single BVP

\[ g(v) \]
Basin boundary in single BVP
Basin boundary in single BVP

Each basin is continuously separable.
Basin boundary of a 2D map

Fractal boundary (Mille-feuille structure) is also separable.

⇒ magnify
Lorenz system

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Stable manifold of the origin forms a “surface.”
Basin boundary ($x$-$y$ plane, with $z = 0$)

$\gamma = 1.3$
Basin boundary \((x-y \text{ plane, with } z = 0)\)

\[ \gamma = 1.238 \]

Boundary is suddenly blurred.
Basin boundary \((x-y\ plane,\ with\ z = 0)\)

\[\gamma = 1.234\]
Basin boundary \((x-y)\) plane, with \(z = 0\)

\[ \gamma = 1.2 \]

not continuously separable.
Fractal basin is not appeared!

\[ \gamma = 1.4 \]
Fractal basin is not appeared!

\[ \gamma = 1.234 \]
Fractal basin is not appeared!

\[ \gamma = 1.238 \]

Boundary is suddenly blurred.
Fractal basin is not appeared!

\[ \gamma = 1.2 \]

God only knows which.
Conclusions

The extended BVP oscillator

analyses of bifurcations and chaos

Jack-in-the-box phenomenon

Blurred basin boundary

Future problems

investigation of the stable manifold of the origin

3D structure of the basin boundary (stable manifold)